

Result! 1) Extension thrm: Every L.I. set in V can be extended to form a basis.

2) Invariance thrm: Any two different basis of V will have same no. of elements.

3) If $\dim V = n$, then any set containing $n+1$ vectors will be L.D.

Eg: Extend $\{ \overset{x_1}{(1, 1, 0, 0)}, \overset{x_2}{(0, 0, 1, 1)} \}$ to form of basis in R^4 .

Let Ex Basis be $\{ x_1, x_2, e_1, e_3, e_3, e_4 \} \Rightarrow$ L.D.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

then $\{ x_1, x_2, e_2, e_4 \}$ is L.I.

Eg $\mathcal{B} = \{ (1, 0, 0, 0), (1, 1, 0, 0) \}$ to a basis of R^4 .

$$U = \{ e_1, x_1 \}$$

n.w! $X = \{ (1, 1, 1, 1) \}$ to form basis in R^4 .

H.W: $X = \{ (1, 1, 1, 1) \}$ or form basis in K .

Result: 4 If U is a subspace of V , then $\dim U \leq \dim V$.

eg: $U = \{ (3, 8, -3, -5), (1, -2, 5, -3), (2, 3, 1, -4) \}$ is a subspace of $\mathbb{R}^4$. Find $\dim U$ & Basis for U .

Result: 5 U_1 & $U_2 \rightarrow$ subspaces

$$\dim(U_1 + U_2) = \dim U_1 + \dim U_2 - \dim(U_1 \cap U_2)$$

RG: A set containing '0' vector is always L.D.

RT: $U_1 + U_2$ & $U_1 \cap U_2$ are subspaces if U_1 & U_2 are subspaces,

but $U_1 \cup U_2$ may or may not be a subspace.

Q: $U = \{ (a, b, c, d) : b + c + d = 0 \}$ $\dim U = 3$

Q: Find $\dim$ & basis of $\mathbb{C} \rightarrow$ complex plane.

Basis of $\mathbb{C} = \{1, i\}$ $\dim = 2 = \dim \mathbb{R}^2$.

H.W: Extract a basis of $\mathbb{R}^3(\mathbb{R})$ from

$\{ (1, -3, 2), (2, 4, 1), (3, 1, 3), (1, 1, 1) \}$